Name: _____

IGCSE (9–1) Maths new topics: Expanding brackets and proof

Date:

Time: 50 minutes

Total marks available: 46

Total marks achieved: _____

[GCSE]

Q1. Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$

for all values of x.

(Total 3 marks)

Q2. Expand and simplify $(x + 5)(x - 3)(x + 3)$

(Total 3 marks)

[GCSE]

Q3. Expand and simplify $(x + 2)(2x - 3)(3x + 1)$

(Total 3 marks)

[GCSE]

Q4. Show that $(x + 1)(x + 2)(x + 3)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are positive integers.

(Total 3 marks)

[GCSE]

Q5. Prove that $(n - 1)^2 + n^2 + (n + 1)^2 = 3n^2 + 2$

(Total 2 marks)

[GCSE]

Q6. Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \text{ is an even number}$$

for all positive integer values of n.

(Total 3 marks)

[GCSE]

Q7. n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

(Total 4 marks)

[GCSE]

Q8. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of 8}$$

for all positive integer values of n.

(Total 3 marks)

[GCSE]

Q9. Prove algebraically that the difference between the squares of any two consecutive integers is always an odd number.

(Total 3 marks)

[GCSE]

Q10. Prove algebraically that the difference between any two different odd numbers is an even number.

(Total 3 marks)

[GCSE]

Q11. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total 4 marks)

[GCSE]

Q12. The product of two consecutive positive integers is added to the larger of the two integers. Prove that the result is always a square number.

(Total 3 marks)

[GCSE]

Q13. Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12.

(Total 3 marks)

[GCSE]

Q14. (*a*) Expand and simplify $x(x + 1)(x - 1)$

(2)

In a list of three consecutive positive integers at least one of the numbers is even and one of the numbers is a multiple of 3.

n is a positive integer greater than 1

(*b*) Prove that $n^3 - n$ is a multiple of 6 for all possible values of n.

(2)

$2^{61} - 1$ is a prime number.

(*c*) Explain why $2^{61} + 1$ is a multiple of 3.

(2)

(Total 6 marks)

Mark Scheme

Q1.

Paper 1MA1: 2H			
Question	**Working**	**Answer**	**Notes**
		Fully correct algebra to show given result	M1 for method to find the product of any two linear expressions; eg. 3 correct terms or 4 terms ignoring signs M1 for method of 6 products, 4 of which are correct (ft their first product) A1 for fully accurate working to give the required result

Q2.

Question	Working	Answer	Mark	AO	Notes
	e.g. $(x^2 + 5x – 3x – 15)(x + 3)$ **or** $(x^2 + 2x – 15)(x + 3)$ **or** $(x – 5)(x^2 + 3x – 3x – 9)$ **or** $(x – 5)(x^2 – 9)$ E.g. $x^3 + 3x^2 + 2x^2 + 6x – 15x – 45$ **or** $x^3 + 5x^2 – 9x – 45$	$x^3 + 5x^2 – 9x – 45$	3	AO1	M1 expansion of any two of the three brackets – at least 3 correct terms M1 (dep) ft for at least 3 correct terms in second expansion A1

Q3.

Mock Paper 1MA1: 3H					
Question		**Answer**	**AO**	**Mark**	**Notes**
		$6x^3 + 5x^2 – 17x – 6$	1.3b	M1	for multiplying out two brackets with at least three terms out of four correct
			1.3b	M1	for a full method
			1.3b	A1	cao

Q4.

Paper: 1MA1/1H				
Question	**Working**	**Answer**	**Mark**	**Notes**
		$x^3+6x^2+11x+6$	M1 M1 A1	for method to find the product of any two linear expressions (3 correct terms) e.g. $x^2+x+2x+2$ or $x^2+2x+3x+6$ or $x^2+x+3x+3$ for method of multiplying out remaining products, half of which are correct (ft their first product) e.g. $x^3+x^2+2x^2+3x^2+2x+3x+6x+6$ cao

Q5.

Question	Working	Answer	Mark	Notes
	$n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$ $3n^2 + 2$	Proof	2	M1 for correct expansion of either $(n-1)^2$ or $(n+1)^2$ or $n^2 - 2n + 1$ or $n^2 + 2n + 1$ seen A1 for $3n^2 + 2$ from correct working

Q6. [part b only]

PAPER: 1MA0_2H					
Question		**Working**	**Answer**	**Mark**	**Notes**
	(a)	$y^2 - 2y - 5y + 10$	$y^2 - 7y + 10$	2	M1 for all 4 terms correct (condone incorrect signs) or 3 out of 4 terms correct with correct signs A1 cao
	*(b)	$(4n^2 + 2n + 2n + 1) - (2n + 1)$ $= 4n^2 + 4n + 1 - 2n - 1$ $= 4n^2 + 2n$ $= 2n(2n + 1)$	Proof	3	M1 for 3 out of 4 terms correct in the expansion of $(2n + 1)^2$ or $(2n + 1)\{(2n + 1) - 1\}$ A1 for $4n^2 + 2n$ or equivalent expression in factorised form C1 for convincing statement using $2n(2n + 1)$ or $2(2n^2 + n)$ or $4n^2 + 2n$ to prove the result

Q7.

Paper: 1MA1/1H				
Question	**Working**	**Answer**	**Mark**	**Notes**
		2(2n−3)	C1	correct expansion of brackets to give at least 3 terms from $n^2-2n-2n+4$
		even	C1	arrives at n^2-2-n^2+4n-4 oe
			C1	reduces to 2(2n−3) or 4n – 6
			C1	for conclusion e.g. 2(2n−3) always even, 4n – 6 is always even since both are even numbers, they are multiples of 2.

Q8.

Question		Working	Answer	Mark	Notes
		$4n^2 + 12n + 3^2 - (4n^2 - 12n + 3^2)$ $= 4n^2 + 12n + 9 - 4n^2 + 12n - 9$ $= 24n$ $=8 \times 3n$	Proof	3	M1 for 3 out of 4 terms correct in expansion of either $(2n + 3)^2$ or $(2n - 3)^2$ **or** $((2n + 3) - (2n - 3))((2n + 3) + (2n - 3))$ A1 for $24n$ from correct expansion of both brackets A1 (dep on A1) for $24n$ is a multiple of 8 or $24n = 8 \times 3n$ or $24n \div 8 = 3n$

Q9.

Mock Paper 1MA1: 2H					
Question		**Answer**	**AO**	**Mark**	**Notes**
		Proof	2.4b	M1	correct expansion or factorisation of a suitable expression for 2 consecutive integers, e.g. $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$ or $(n + 1)^2 - n^2 = (n + 1 + n)(n + 1 - n)$
			2.4b	A1	expansion or factorisation correctly simplified, e.g. $2n + 1$ or $2n + 3$
			2.4b	C1	correct conclusion drawn from fully correct working

Q10.

Mock Paper 1MA1: 1H					
Question		Working	Answer	Mark	Notes
			Proof	3	M1 for expressions to represent any 2 different odd numbers, e.g. $2n+1$ and $2m+1$ M1 for method to subtract and factorise. C1 correct expression and conclusion

Q11.

		Working	Answer	Mark	Notes
*		$(n+1)^2 - n^2$ $= n^2 + 2n + 1 - n^2$ $= 2n + 1$ $(n + 1) + n = 2n + 1$ OR $(n + 1)^2 - n^2$ $= (n + 1 + n)(n + 1 - n)$ $= (2n + 1)(1) = 2n + 1$ $(n + 1) + n = 2n + 1$ OR $n^2 - (n + 1)^2 = n^2 - (n^2 + 2n + 1) =$ $-2n - 1 = -(2n + 1)$ Difference is $2n + 1$ $(n + 1) + n = 2n + 1$	proof	4	M1 for any two consecutive integers expressed algebraically eg n and $n+1$ M1(dep on M1) for the difference between the squares of 'two consecutive integers' expressed algebraically eg $(n + 1)^2 - n^2$ A1 for correct expansion and simplification of difference of squares, eg $2n + 1$ C1 (dep on M2A1) for showing statement is correct, eg $n + n + 1 = 2n + 1$ and $(n + 1)^2 - n^2 = 2n + 1$ from correct supporting algebra

Q12.

Question	Working	Answer	Notes
		proof	C1 starts proof eg $n(n+1)$ or $(n-1)\times n$ C1 $n(n+1) + n+1$ or $(n-1)\times n + n$ C1 for convincing proof including $(n+1)^2$ or n^2

Q13.

Mock Paper 1MA1: 1H					
Question		Answer	AO	Mark	Notes
19		Shows result	2.4b	C1	shows expansion; for example, $(4n^2 + 4n + 1)$ or $(4n^2 + 12n + 9)$ or $(4n^2 + 20n + 25)$
			2.4b	C1	simplifies; for example, $12n^2 + 36n + 35$
			2.4b	C1	arrives at $12(n^2 + 3n + 2) + 11$ and concludes result

Q14.

Question	Working	Answer	Mark	AO	Notes
(a)		Shown	M	1.3a	M1 for $x(x^2 - 1)$ or $(x^2 + x)(x - 1)$ oe
			A	1.3a	A1 cao
(b)		Shown	P	2.4b	P1 for explanation to show that $n^3 - n$ is the product of three consecutive positive integers, e.g. $n^3 - n = (n-1)n(n+1)$
			C	2.4b	C1 for a correct conclusion to the proof, e.g. at least one of these is even and one is a multiple of 3 so the product is a multiple of 6
(c)	$2^{61} - 1$ is prime so not a multiple of 3 2^{61} is not a multiple of 3 Hence $2^{61} + 1$ must be a multiple of 3	Shown	P	2.4a	P1 for recognising that $2^{61} - 1$, 2^{61} and $2^{61} + 1$ are three consecutive positive integers
			C	2.4a	C1 for a convincing argument